2019 American Control Conference (ACC)
Philadelphia, PA, USA, July 10-12, 2019

Extremum seeking for real-time optimal drilling control*

Ulf Jakob F. Aarsnes[1], Ole Morten Aamo[2] and Miroslav Krstic[3]

Abstract— **This paper demonstrate the feasibility and illustrates some challenges of applying extremum seeking control to online optimization of the drilling process. Specifically, we consider the problem of finding the hook-load (and consequently the weight on bit) which optimizes the rate of penetration while drilling. To this end, a phenomenological drilling model is presented which includes the bit foundering that occurs at too high weight on bit. We then propose an Extremum Seeking (ES) controller architecture which can be used in conjunction with existing auto-driller systems. The effectiveness of this ES architecture is illustrated by simulation examples with the presented model.**

I. Introduction

The cost of drilling a well is in large part determined by the time it takes to drill it. Hence, cost-reduction is achieved through reducing Non-Productive Time (NPT) and increasing Rate of Penetration (ROP). We will focus on increasing ROP.

The basic mechanism of the cutting process is quite well understood in the ideal case [1], [2], [3]. However, the relevance of findings from such idealized models towards the goal of optimizing ROP is limited in the field situation, due to the large uncertainty and significant complexity of downhole dynamics in drilling. This have led to Black Box approaches: attempts to analyzs field data to find correlations between operating parameters and ROP, or in drilling parlance, to identify operational sweet spots. However, this form of big-data approach have had limited success due to the multitude of changing operating conditions reducing the applicability from one case to the next [4]. A more promising approach is to optimize drilling conditions online based on feedback from real-time measurements. Such an approach is allowed by Extremum Seeking Control [5].

The drilling optimization problem can be thought of as maximizing the Rate Of Penetration (ROP) by tweaking certain operating parameters subject to constraints imposed by rig equipment and various other qualitative considerations decided by the driller. While increasing the drill string Revolutions Per Minute (RPM) typically yields a monotonic increase in ROP and is constrained by the possible occurrence of whirl, increasing Weight On Bit (WOB) only leads to an increase in ROP up until the foundering point [6], [7], after which the mechanical energy is wasted and damage to the Poly Crystaline Cutters (PDC) can occur. In [2] the cutting process of a PDC bit is split into three successive regimes: Phase I is dominated by frictional contact process due to bit dullness. Phase II, where the contact forces are fully mobilized and additional forces contribute to cutting. Phase III, where the frictional contact forces increase due to insufficient hole cleaning see Fig. 3. The foundering point is then located at the Phase II/III transition, and drilling should ideally be performed close to the foundering point [6], [7]. However, as is noted by [2]:

> There is reason to believe that this mode of operation is rarely achieved in practice. Indeed, analysis of field data indicates that the drilling efficiency is typically low, thus suggesting that the bit is not often drilling in phase II [and] (in the field environment) phase II can be missed all together; furthermore the characteristic contact length l (one of the parameters controlling w_f, the weight-on-bit at the transition phase I/phase II) is also changing constantly due to wear and self-sharpening of the bit.

Hence, to achieve optimal drilling performance, weight on bit should be continuously updated in real-time to adapt to the changing operating conditions, based on feedback from the drilling data.

Extremum seeking is a non-model based approach to real-time optimization in situations where there is a nonlinearity with a local extremum in the control problem [5]. It has been in existence since the 1950s The proof of convergence to a local extremum for a general class of non-linear systems is shown in [8]. It has been widely applied to industrial problems [5], [9], including for petroleum production [10], [11]. Its application in drilling have very recently also been proposed [12], and its feasability is evaluated in this paper.

II. Introduction to extremum seeking

We now give a brief introduction to extremum seeking control. We consider a general single input–single output (SISO) nonlinear model, following [9]:

$$\dot{x} = f(x, \theta) \tag{1}$$
$$y = h(x, \theta) \tag{2}$$

where $x \in \mathbb{R}^n$ is the state, $\theta \in \mathbb{R}$ is the input, $y \in \mathbb{R}$ is the output, and $f : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$. Assume that there exists a smooth function $l : \mathbb{R} \to \mathbb{R}^n$ such that $l(\theta)$ is an exponentially stable equilibrium point of the system (1)–(2),

The work of the first author was supported by the Research Council of Norway, ConocoPhillips, AkerBP, Statoil and Wintershall through the research center DrillWell (203525/O30) at IRIS, and by the Marie Skłodowska-Curie (MSCA) - Individual Fellowships through the Robust Estimation and Control of Infinite Dimensional Systems (RECIDS) Project.

[1]Norwegian Research Centre, Essendropsgate 3, Oslo, Norway. ulaa@norceresearch.no

[2]Department of Engineering Cybernetics, Norwegian University of Science and Technology, Trondheim, Norway.

[3]Department of Mechanical and Aerospace Engineering, University of California, San Diego, La Jolla, USA.

978-1-5386-7926-5/$31.00 ©2019 AACC

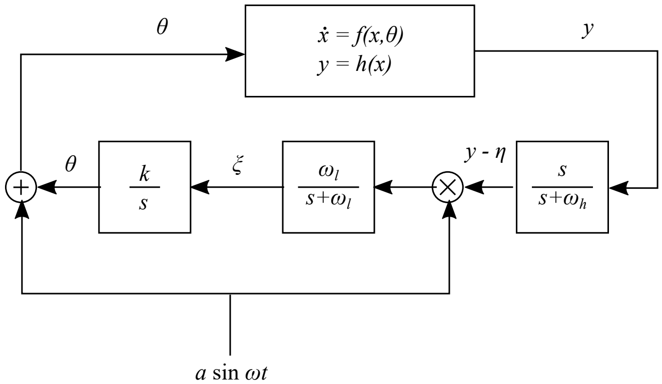

Fig. 1. Basic extremum seeking scheme.

that is

$$f(l(\theta), \theta) = 0. \tag{3}$$

Further assume that there exists $\theta^* \in \mathbb{R}$ such that

$$(h \circ l)'(\theta^*) = 0 \tag{4}$$
$$(h \circ l)''(\theta^*) < 0. \tag{5}$$

Hence we assume that the output equilibrium map $y = h(l(\theta))$ has a *maximum* at $\theta = \theta^*$. Our goal is to develop a feedback mechanism which approaches this optimal value without requiring knowledge of either θ^* or the system f, h, l. The feedback scheme is shown in Fig. 1. The basic idea of this approach employs a slow periodic perturbation $a \sin \omega t$ which is added to the signal $\hat{\theta}$, our current best estimate of θ^*. For a slow perturbation, relative to the plant response, then the plant appears as a static map $y = h \circ l(\theta)$ and its dynamics do not interfere with the extremum seeking scheme. The perturbation in θ will create a periodic response in y which is either in phase or out of phase with $a \sin \omega t$. The high-pass filter $s/(s + \omega_h)$, known as the "washout filter", eliminates the "dc component" of y. Thus, the perturbation signal $a \sin \omega t$ and $(s/(s + \omega_h))y$ approximates two sinusoids which are

- in phase for $\hat{\theta} < \theta^*$
- out of phase for $\hat{\theta} > \theta^*$.

The product of these two signals have a "dc component", ξ which is extracted by the low-pass filter and then integrated to obtain the estimate $\hat{\theta}$. The overall feedback system has three time scales:

- Fastest – the plant response.
- Medium – the periodic perturbation.
- Slow – the filters in the extremum seeking scheme.

III. Drilling model

We will apply the peak seeking scheme to a phenomenological model describing the axial dynamics of the drilling system. We use a semi-lumped description, conceptually similar to [13], based on the distributed drill-string model as derived in [14], [15].

A. Distributed dynamics

Denote the axial velocity and force by $v(t, x), w(t, x)$, respectively, where $(t, x) \in [0, \infty) \times [0, L]$, with L the length of the drill-string. The axial force can be found from the strain, given as the local relative compression $w(t, x) = AE\big(\xi(t, x) - \xi(t, x{+}dx)\big)/dx$, where $\xi(t, x)$ is the axial displacement such that $\frac{\partial \xi(t,x)}{\partial t} = v(t, x)$, and $dx \to 0$ is an infinitesimal axial position increment, A is the cross-sectional area of the element and E is the Young's modulus. Further, ρ is the pipe mass density and k_a is a damping coefficient representing the viscous shear stresses acting on the pipe, and $\frac{\bar{\rho}}{\rho} g \sin \phi(x)$ accounts for the acceleration of gravity acting on the submerged weight $\bar{\rho}$. Then, the axial motion is governed by the 1-D wave equation with viscous damping:

$$\frac{\partial w(t,x)}{\partial t} + AE \frac{\partial v(t,x)}{\partial x} = 0 \tag{6}$$
$$\frac{\partial v(t,x)}{\partial t} + \frac{1}{A\rho} \frac{\partial w(t,x)}{\partial x} = -k_a v(t,x) + \frac{\bar{\rho}}{\rho} g \sin \phi(x). \tag{7}$$

The topside velocity $v(x = 0, t)$ is the system actuation, and the downhole boundary condition at $x = L$ is obtained from a force balance on the lumped Bottom-Hole Assembly (BHA) with mass M_b. Both of these will be discussed in the following.

B. Topside boundary

In a drilling system, the drill string is connected at the top to the *top-drive* suspended over the drill floor by the *traveling block*. The traveling block is connected by several steel *drill lines* with one attached to the deadline anchor and the other being spooled on a drum controlled by AC induction motors [16]. Thus, the spool-rate (or "feed-rate") of this AC motor controls the axial velocity of the traveling block $v_0(t)$. Hence, at the left boundary (topside) we have the boundary condition

$$v(t, x = 0) = v_0(t). \tag{8}$$

The measured output of the system is the strain at the deadline anchor, which is used to compute the weight as felt by the hoisting system which corresponds to the axial force acting from the top-drive on the drill string [17], in drilling parlance referred to as the *hook load*. We will denote the Hook load as $w_0 := -w(t, x{=}0)$. Roughly speaking, the hook load represents the suspended weight of the drill string minus the weight on bit. Due to lack of down-hole measurements, the hook load is in practice used as a direct proxy for weight on bit during drilling operations.

To maintain the right weight on bit, the feed-rate is often set by PI-feedback controller from the hook load called an *Auto-driller* [18], where a set point w_0^{sp} for the desired hook load is provided by the driller:

$$v_0(t) = K_p(w_0(t) - w_0^{sp}) + K_i \int_0^t (w_0(\tau) - w_0^{sp}) \mathrm{d}\tau. \tag{9}$$

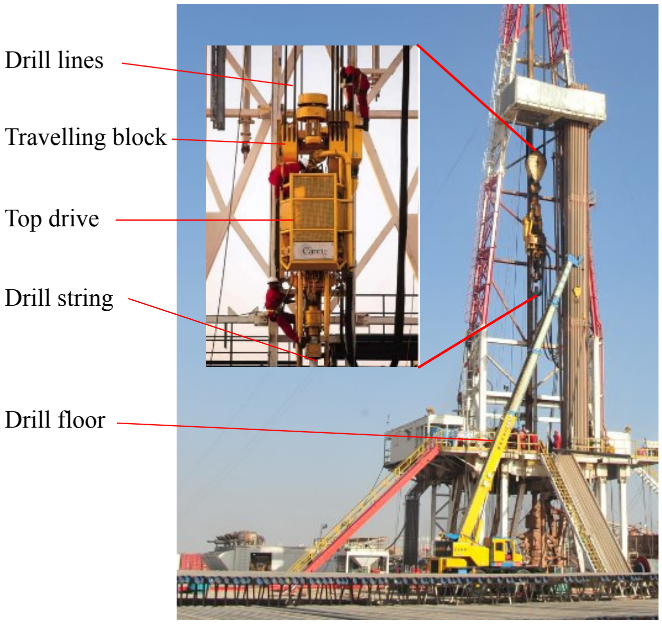

Fig. 2. Picture showing the topside of a drilling rig.

C. Bottom-hole boundary

The bottom-hole boundary condition at $x = L$ is obtained from a force balance on the lumped Bottom-Hole Assembly (BHA) with mass M_b. We will denote the velocity at the bottom-hole boundary as $v(t, x = L) =: v_b(t)$, and the force acting from the drill string on the BHA as $w(t, x{=}L) =: w_L(t)$. The bottom-hole boundary condition then writes as:

$$M_b \dot{v}_b = w_b(v_b, w_L) - w_L + \frac{\bar{\rho}}{\rho} M_b g, \tag{10}$$

where $w_b(v_b, w(t, x{=}L))$ is the force acting from the formation on the BHA through the drilling bit, known as the *weight on bit*. Following [16] we can relate the weight on bit to the bit velocity by considering the *combined depth of cut* [19] per revolution $d(t) = \frac{v_b(t)}{\omega_{\text{bit}}}$, where ω_{bit} denote revolutions per second of the drill bit, and then relating combined depth of cut to weight on bit, see Fig. 3. Note that we have in the current investigation assumed constant bit angular velocity ω_{bit}. Following [2], the cutting process is decomposed into three phases. We implement this in the following way, where the five cases correspond to: bit off bottom, Phase I, Phase II, and Phase III.

$$w_b = \begin{cases} 0, & d < 0 \\ w_L, & d = 0 \text{ and } w_L \le w_{f*} \\ w_{f*} + K_a d, & d_b > d \ge 0 \text{ and } w_L > w_{f*} \\ w_{f*} + K_a d_b, & d \ge d_b \text{ and } w_L < w_{f*} + K_a d_b \\ w_L, & d \ge d_b \text{ and } w_L > w_{f*} + K_a d_b \end{cases} \tag{11}$$

where w_{f*} denotes the weight on bit at the Phase I/II transition, and d_b denotes the *combined* depth of cut at the Phase II/III transition. Finally, $K_a = \zeta \epsilon a$, where ζ characterizes the cutting angle, ϵ the intrinsic specific energy and a the bit radius [19], [1].

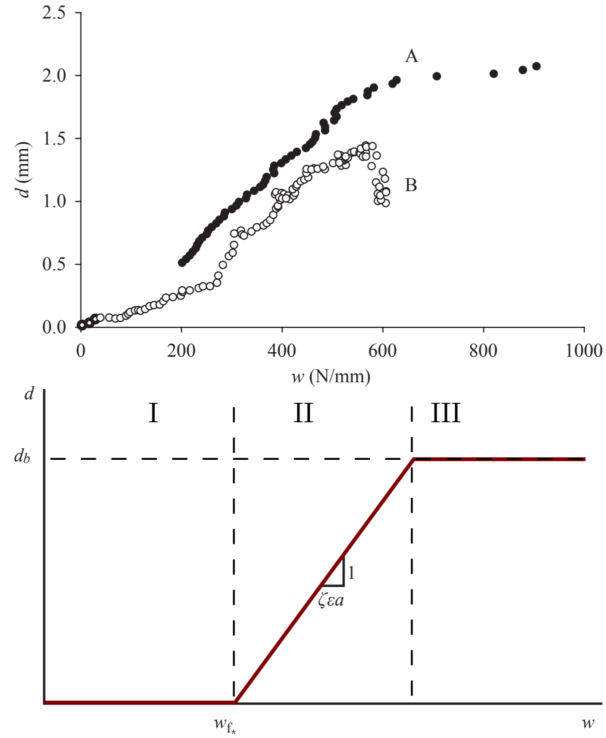

Fig. 3. Top: depth of cut, d, (corresponding to a ROP) for different WOB, w. From experiments with A: Kinematic control, and B: weigh-on-bit control (due to [2]). Bottom: Conceptual schematic of the same relation as used in the present note with the three Phases indicated.

TABLE I
DRILLING SYSTEM PARAMETERS

A	$3.5 * 10^{-3}$ m^2	d_b	$8 * 10^{-3}$ m
E	$2 * 10^{11}$ Pa	k_a	0.1 1/s
K_a	$3.88 * 10^6$ N/m	L	2000 m
M_b	12000 kg	M_{HW}	4034.7 kg
w_{f*}	71280 N	ρ	8000 kg/m^3
$\bar{\rho}$	7000 kg/m^3	ω_{bit}	1.05 1/s

D. Example simulation

The drilling model described in this section is simulated with the parameters given in Table I. The drilling system uses an auto-driller with the $K_p = 4.6 * 10^{-7}$ m/(sN), $K_i = 7 * 10^{-8}$ m/(s^{2}N). We consider a drilling system in Phase I, and then lower the set point of the hook load in steps such that the system enters Phase II and then Phase III, see Fig. 4. The drilling system, with the auto-driller, has a closed loop response time which is primarily given by the aggressiveness of the auto-driller tuning and rock properties. However, the aggressiveness of the auto-driller tuning is dependent on length and geometry of the drill string (the delay and BHA time constant). These points are worthwhile noting as this time constant determines the convergence speed of the extremum seeking algorithm we propose in the following.

IV. Extremum seeking control of drilling

The simplest way to implement Extremum seeking control with current drilling control systems is to use it in cascade

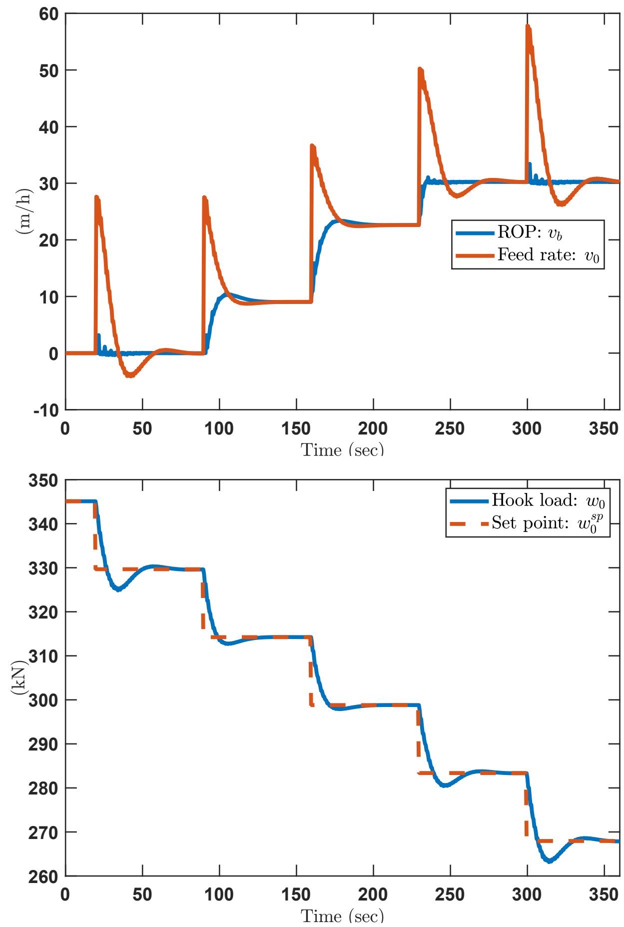

Fig. 4. Simulated drilling response to step changes in the hook load set point.

with existing auto-drillers. This approach is schematically depicted in Fig. 5.

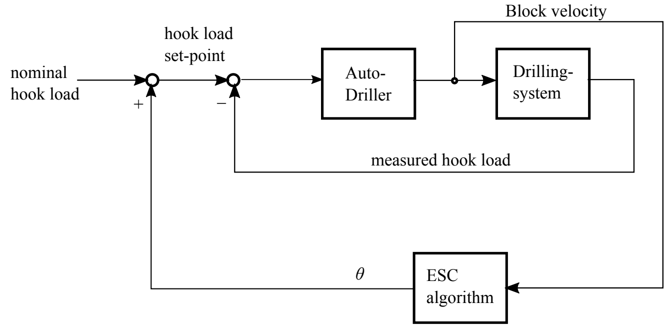

Fig. 5. Proposed ESC scheme with auto-driller.

A. ESC algorithm

We implement a basic extremum seeking control procedure to enable online configuration of the optimal hook load set-point, w_0^{sp}, for the auto-driller. The implementation is described in the following. Our goal is for the ESC to control the drilling system to the Phase II/III transition (the foundering point), which is often considered to be the optimal point of operation [6]. The input is the set-point $w_0^{sp}(t) = w_0^{\text{nom}} + \theta(t)$, where w_0^{nom} is a initial nominal hook load set by the driller and $\theta(t)$ is governed by Extremum Seeking Control (ESC) algorithm. To satisfy the assumptions (4)-(5) at this point, the system output is defined to be

$$y = v_0 - k_\theta \theta, \tag{12}$$

where $0 < k_\theta < K_a \omega_{\text{bit}}$. We use the following Extremum Seeking Control (ESC) algorithm [5]:

$$\theta(t) = \hat{\theta}(t) + M(t) \tag{13}$$

$$\dot{\hat{\theta}}(t) = K\hat{G}(t) \tag{14}$$

$$M(t) = a\sin(\omega t), \tag{15}$$

where $M(t)$ is the perturbation signal, K is the gain and the filtered signals

$$\hat{G}(s) = \frac{\omega_l}{s + \omega_l} M z \tag{16}$$

$$z = \frac{s}{s + \omega_h} y. \tag{17}$$

B. Static map

Denote the states at the equilibrium by an overbar, i.e. $\bar{w}, \bar{v}$, etc. We find the steady state by setting the time derivatives in (6),(7),(10) to zero, and noting that the integral term in (9) ensures that $\bar{w}_0 = \bar{w}_0^{sp}$. We have

$$\bar{w}_b = M_{HW} g - \bar{w}_0^{sp} - \bar{v}_0 k_v, \tag{18}$$

where the drill string *hanging weight* is given as

$$M_{HW} = \int_0^L A\bar{\rho} \sin\phi(x) \mathrm{d}x + \frac{\bar{\rho}}{\rho} M_b, \tag{19}$$

and the viscous damping coefficient $k_v = LA\rho k_a$. Combining with (11) we obtain, when the system is at equilibrium, the *static map*

$$y(\theta) = \begin{cases} -k_\theta \theta, & \text{Phase I} \\ \frac{\omega_{\text{bit}} - K_a k_\theta}{K_a + \omega_{\text{bit}} k_v} \theta + K_y, & \text{Phase II} \\ -k_\theta \theta + d_b \omega_{\text{bit}}, & \text{Phase III} \end{cases} \tag{20}$$

where $K_y = \frac{\omega_{\text{bit}}}{K_a + \omega_{\text{bit}} k_v} (M_{HW} g - w_0^{nom} - w_{f*})$. Assuming that $\omega_{\text{bit}} - K_a k_\theta$, then this static map is convex upward, and has the Phase II/III transition as a peak, in the interval comprised of Phase II and Phase III. Hence, we obtain the following requirements for the success of the ESC algorithm:

1) $0 < k_\theta < \frac{\omega_{\text{bit}}}{K_a}$
2) $w_0^{nom} < M_{HW} g - w_{f*}$

Note that this static map does not satisfy the assumptions given in Section II, specifically, it is not smooth. There are two points to be made regarding this: Firstly, the assumption of smoothness of the static map is in some cases overly stringent. As we shall see in the simulations presented in the following, it is for the present system sufficient that the static map is convex upward and that 0 is contained in the *subdifferential* at the peak. Secondly, we could have approximated the static map with a smooth map satisfying the assumptions in Section II and still have a good representation of the weight on bit seen in experiments, see Fig. 3.

C. Filter design

In setting the filter parameters, we use the suggestions from [9]: $\omega_h = \omega$, $\omega_l = \omega/3$. The period of the periodic perturbation ω should be slower than the plant response, and based on the simulations shown in Fig. 4, we choose $\omega = 0.1$ rad/s, which corresponds to a period of 62 seconds. However, for longer wells or less aggressive auto-drillers, a slower perturbation signal might be required. Finally, we use the perturbation amplitude $a = 3000$ N, and the gain $K = 200$ m^{-1}.

D. Simulation results

In the simulations we consider a drilling system with the parameters given in Table I. The drilling system uses an auto-driller with the $K_p = 4.6 * 10^{-7}$ m/(sN), $K_i = 7 * 10^{-8}$ m/(s^{2}N). The simulation results are shown in Fig. 6 for two different runs.

1) Run 1: For run 1 the the drilling system is close to the Phase I/II transition, when the ESC is started at $t = 120$ sec. The ESC algorithm effectively controls the system to the optimal operating point at the Phase II/III transition over the next 10 minutes. In Fig. 6, bottom, the WOB can be seen to be increased gradually until the ROP reaches its maximum (Fig. 6, top).

2) Run 2: For run 2, a perhaps more realistic scenario is considered. Here the the drilling system is operating at a significantly elevated weight on bit which does not contribute to ROP but increases the risk of vibrations and bit damage. The ESC is started at $t = 120$ sec, and the algorithm gradually increases the hook load, corresponding to an decrease in weight on bit (Fig. 6, bottom), until a decrease in ROP occurs (Fig. 6 top). Thereby, the system is effectively controlled to the optimal operating point at the Phase II/III transition.

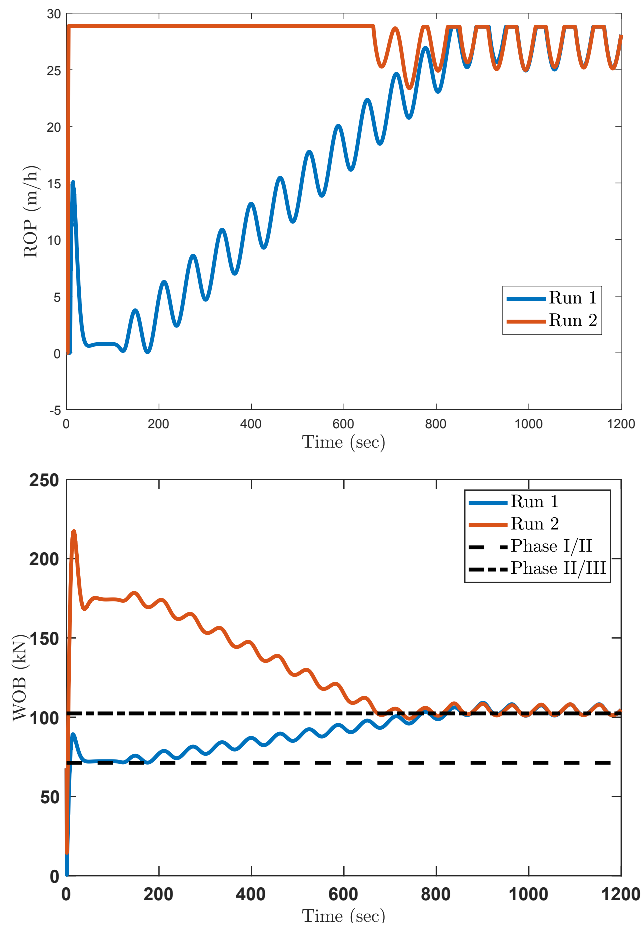

Fig. 6. Drilling response for two runs with extremum seeking control of $w_0^{sp}(t) = w_0^{\text{nom}} + \theta(t)$ which effectively finds the hook load $w_0(t)$ corresponding to the optimal WOB at the phase II/III transition. The extremum seeking is started at 120 seconds.

V. Discussion and Perspectives

In this paper we have demonstrated the feasibility of applying extremum seeking control of the hook load to optimize ROP while drilling. The key point that enables such an approach is the concept of bit foundering. That is, the fact that ROP tapers off (and sometimes starts decreasing) with increasing weight on bit past the foundering point. This makes the static mapping between ROP and weight on bit upwards convex in an interval around the foundering point, i.e. the transition between the phase II and phase III drilling regime [2]. As was shown in this paper, using an auto-driller with integral action, this transfers to an upwards convex static mapping between the equilibrium hook load set point and feed rate. Consequently, these can be used as, respectively, the plant input and output for the design of an extremum seeking control scheme.

The requirements for this scheme to work can be summarized as:

- Perturbation radial velocity ω small enough relative to system time constant.
- Penalty coefficient satisfying: $0 < k_\theta < K_a \omega_{\text{bit}}$
- The ESC is started with the system in Phase II or Phase III of drilling: $w_0^{nom} < M_{HW} g - w_{f*}$.

A further challenge is the possible occurrence of complex drill-string phenomena, not represented in the current model. Potential disturbing dynamics could be drill string vibrations, or coupling of the axial dynamics with torsional and/or lateral movement, that cause transient phenomena with larger time constants. If such transient dynamics are evident, the perturbation period might have to be increased further.

This paper's goal has only been to illustrate the feasibility and potential of applying ESC to drilling control, and as such has limited the design to a low complexity, easily applicable, formulation. There are several aspects that could be improved or extended:

- Improving/changing the auto-driller to reduce closed loop time constant.
- Improve performance by explicitly taking the drill string delay dynamics into account [20].
- Simultaneously optimize over multiple drilling parameters [21].
- Optimize over other parameters than ROP, such as minimizing MSE [7].

REFERENCES

[1] E. Detournay and P. Defourny, "A phenomenological model for the drilling action of drag bits," *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts*, vol. 29, no. 1, pp. 13–23, 1992.

[2] E. Detournay, T. Richard, and M. Shepherd, "Drilling response of drag bits: Theory and experiment," *International Journal of Rock Mechanics and Mining Sciences*, vol. 45, pp. 1347–1360, dec 2008.

[3] Y. Zhou and E. Detournay, "Analysis of the contact forces on a blunt PDC bit," in *48th US Rock Mechanics / Geomechanics Symposium 2014*, 2014.

[4] E. Maidla, W. Maidla, J. Rigg, M. Crumrine, and P. Wolf-Zoellner, "Drilling Analysis Using Big Data has been Misused and Abused," in *IADC/SPE Drilling Conference and Exhibition*, Society of Petroleum Engineers, mar 2018.

[5] K. B. Ariyur and M. Krstić, *Real-Time Optimization by Extremum-Seeking Control*. John Wiley & Sons, Inc., 2003.

[6] *The IADC Drilling Manual*. INTERNATIONAL ASSOCIATION OF DRILLING CONTRACTORS, 12th ed., 2014.

[7] F. E. Dupriest and W. L. Koederitz, "Maximizing Drill Rates with Real-Time Surveillance of Mechanical Specific Energy," *SPE/IADC Drilling Conference*, 2005.

[8] M. Krstić and H.-H. Wang, "Stability of Extremum Seeking Feedback for General Nonlinear Dynamic Systems," *Automatica*, vol. 36, pp. 595–601, 2000.

[9] Hsin-Hsiung Wang, S. Yeung, and M. Krstic, "Experimental application of extremum seeking on an axial-flow compressor," *IEEE Transactions on Control Systems Technology*, vol. 8, pp. 300–309, mar 2000.

[10] A. Pavlov, M. Haring, and K. Fjalestad, "Practical extremum-seeking control for gas-lifted oil production," *2017 56th IEEE Conference on Decision and Control (CDC)*, no. Cdc, pp. 2102–2107, 2017.

[11] J. A. Peixoto, D. Pereira-dias, A. F. S. Xaud, and A. R. Secchi, "Modelling and Extremum Seeking Control of Gas Lifted Oil Wells," in *IFAC-PapersOnLine*, vol. 48, pp. 21–26, Elsevier Ltd., 2015.

[12] C.-S. J. Ng and S. Khromov, "Method and system for performing automated drilling of a wellbore," 2019.

[13] U. J. F. Aarsnes, F. Di Meglio, and R. J. Shor, "Avoiding stick slip vibrations in drilling through startup trajectory design," *Journal of Process Control*, vol. 70, pp. 24–35, oct 2018.

[14] C. Germay, V. Denoël, and E. Detournay, "Multiple mode analysis of the self-excited vibrations of rotary drilling systems," *Journal of Sound and Vibration*, vol. 325, pp. 362–381, aug 2009.

[15] U. J. F. Aarsnes and N. van de Wouw, "Axial and torsional self-excited vibrations of a distributed drill-string," *Journal of Sound and Vibration*, vol. 444, no. March, pp. 127–151, 2019.

[16] E. Cayeux, "On the Importance of Boundary Conditions for Real-Time Transient Drill-String Mechanical Estimations," in *IADC/SPE Drilling Conference and Exhibition*, Society of Petroleum Engineers, mar 2018.

[17] E. Cayeux, B. Daireaux, E. W. Dvergsnes, and F. Florence, "Toward Drilling Automation: On the Necessity of Using Sensors That Relate to Physical Models," *SPE Drilling & Completion*, vol. 29, pp. 236–255, SPE–163440–PA, mar 2014.

[18] G. Boyadjieff, D. Murray, A. Orr, M. Porche, and P. Thompson, "Design Considerations and Field Performance of an Advanced Automatic Driller," in *SPE/IADC Drilling Conference*, Society of Petroleum Engineers, apr 2003.

[19] T. Richard, C. Germay, and E. Detournay, "A simplified model to explore the root cause of stickslip vibrations in drilling systems with drag bits," *Journal of Sound and Vibration*, vol. 305, pp. 432–456, aug 2007.

[20] T. R. Oliveira, M. Krsti, and D. Tsubakino, "Extremum Seeking for Static Maps With Delays," *IEEE Transactions on Automatic Control*, vol. 62, no. 4, pp. 1911–1926, 2017.

[21] A. Ghaffari, M. Krstić, and D. Nešić, "Multivariable Newton-based extremum seeking," *Automatica*, vol. 48, no. 8, pp. 1759–1767, 2012.