Imbibition with solidification in alumina feeding

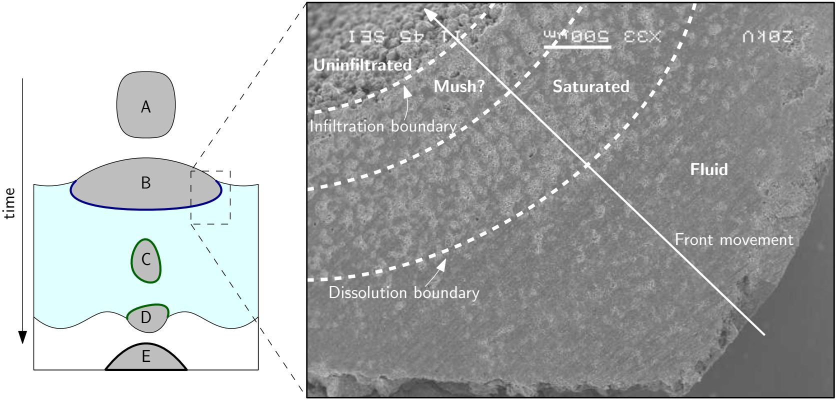
Attila Kovacs
InFoMM CDT, Mathematical Institute, University of Oxford

Chris Breward, James Oliver and Andreas Münch (Oxford)
Svenn Anton Halvorsen and Ellen Nordgård-Hansen (NORCE)
Eirik Manger (Hydro Aluminium)

APS-DFD (Seattle) 2019

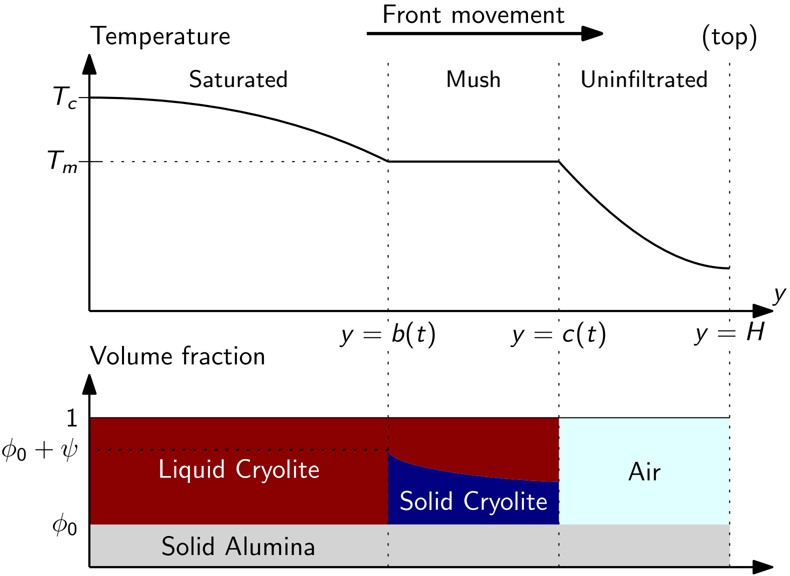
November 25, 2019

Alumina feeding: raft problem



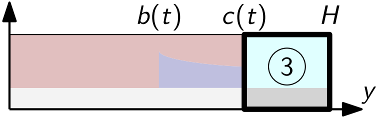
Source: [Kaszas2016]

1D Infiltration problem



Similar models e.g. [Mortensen1989]

1D Infiltration problem



Uninfiltrated

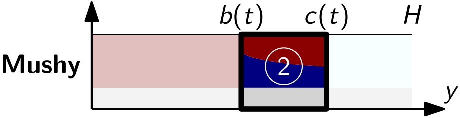
For $c(t) < y < H$:

$$\frac{\partial T_3}{\partial t} = \frac{\partial^2 T_3}{\partial y^2},$$

$$\text{At} \quad y = c(t): \quad T_3 = 1 - \varepsilon, \quad \psi \dot{c} = -\mathrm{St} k_{pf} \frac{\partial T_3}{\partial y},$$

$$\text{At} \quad y = 1: \quad \frac{\partial T_3}{\partial y} = \mathrm{Nu}(T - \theta_e)$$

1D Infiltration problem



For $b(t) < y < c(t)$:

$$\frac{\partial \psi}{\partial t} = 0,$$

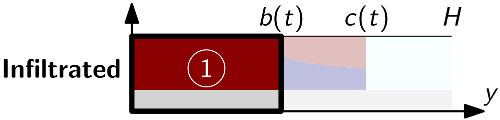
$$\frac{\partial}{\partial y}\left((1-\phi-\psi)u_2\right) = 0,$$

$$-\gamma \frac{K(\psi+\phi)}{(1-\phi-\psi)}\left(\frac{\partial p_2}{\partial y} + \beta\right) = u_2,$$

At $y = b(t)$: $[p_i]_-^+ = 0, \quad u_1 - \frac{1-\phi-\psi}{1-\phi}u_2 = \psi \dot{b}\frac{1-\rho}{1-\phi},$

At $y = c(t)$: $p_2 = -1, \quad u_2 = \dot{c}\left(1 + \frac{\psi\rho}{1-\phi-\psi}\right),$

1D Infiltration problem



For $0 < y < b(t)$:

$$\frac{\partial T_1}{\partial t} + \frac{\partial}{\partial y}\left(q\phi_c u_1 T_1\right) = \alpha_{ip}\frac{\partial^2 T_1}{\partial y^2},$$

$$\frac{\partial}{\partial y}\left((1-\phi)u_1\right) = 0,$$

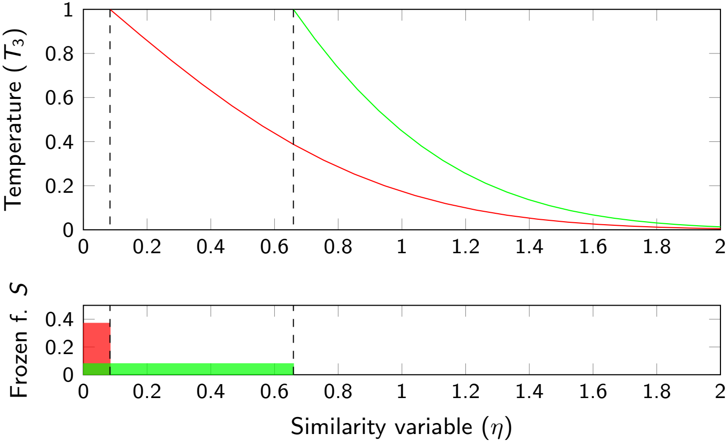
$$\gamma\frac{-K(\phi)}{(1-\psi)}\left(\frac{\partial p_1}{\partial y} + \beta\right) = u_1,$$

$$\text{At} \quad y = 0: \quad T_1 = 1, \quad p_1 = 0,$$

$$\text{At} \quad y = b(t): \quad T_1 = 1 - \varepsilon, \quad \psi\dot{b} = -\mathrm{St}k_{if}\frac{\partial T_1}{\partial y},$$

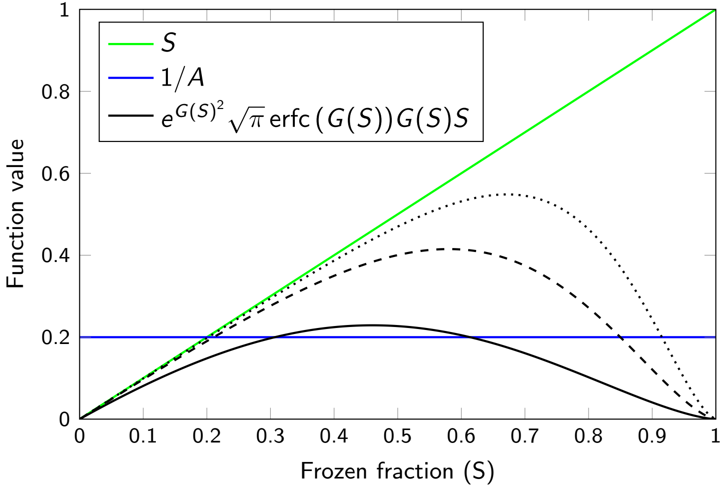
$$[p_i]_-^+ = 0, \quad u_1 - \frac{1-\phi-\psi}{1-\phi}u_2 = \psi\dot{b}\frac{1-\rho}{1-\phi},$$

Similarity solutions for $A = 10$, $\gamma = 10$



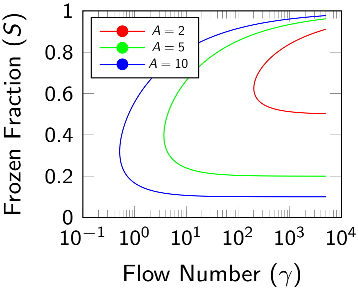
For a parameter set two different solutions are possible: faster propagating with less freeze (🟩) or slower moving with more freeze (🟥). [Tsypkin2005]

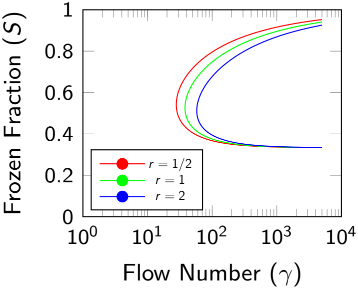
Reason for multiple solutions

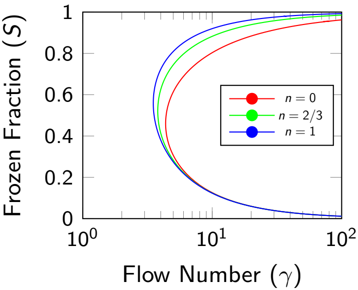


Nonlinear system splits into“fluid”(■) and “solidification” (🟦) parts.

Parameter dependence

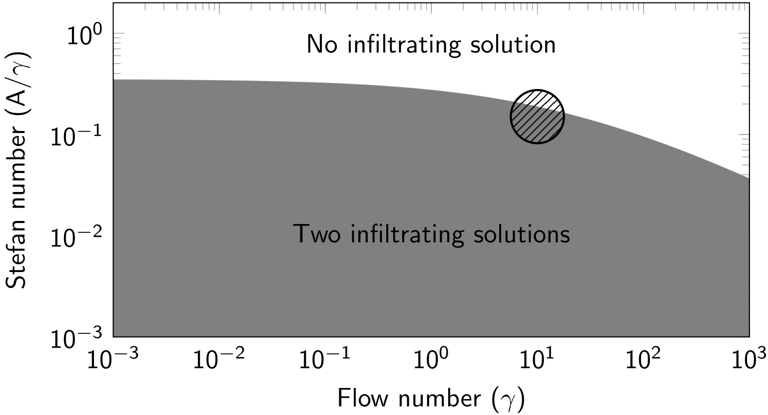






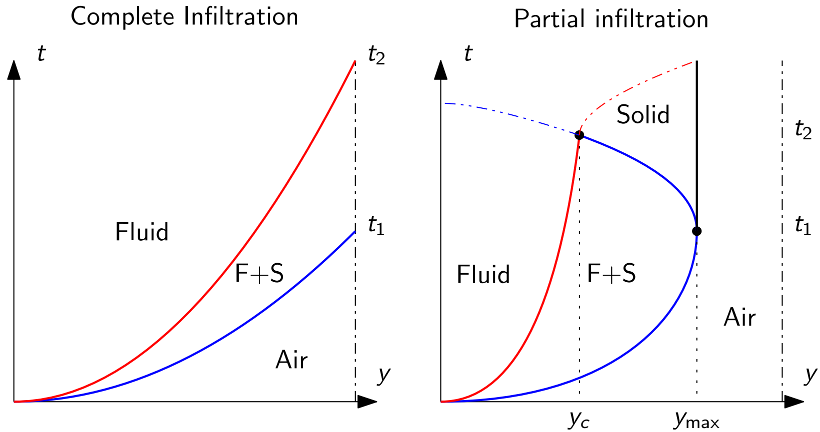
Changing the density slightly changes the necessary pressure needed for infiltration, the form of the capillary pressure function changes the behaviour only at large frozen fractions and A has large effect on the solution.

Regime diagram for early time solutions



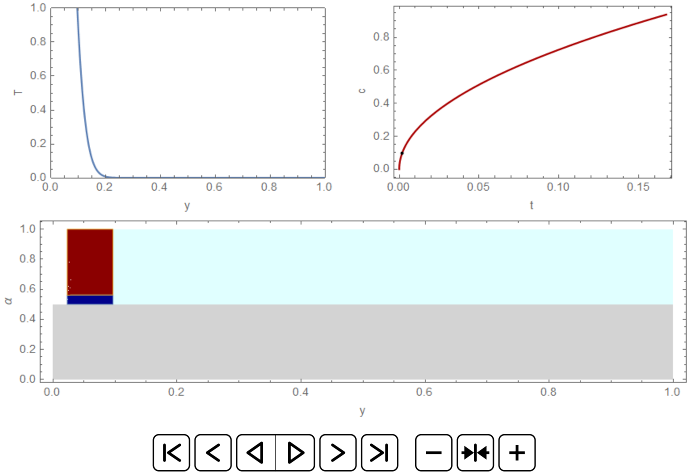
Changing the parameters of the operating regime (dashed) can change the behaviour that we would expect happening.

Further stages



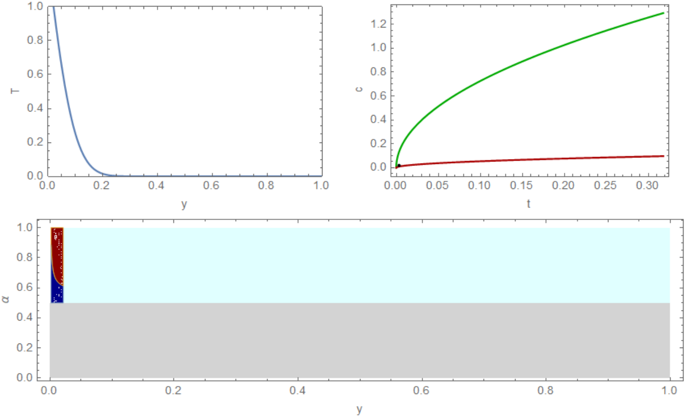
Depending on the boundary condition at the top, the raft can either get fully infiltrated or partially.

Numerical results



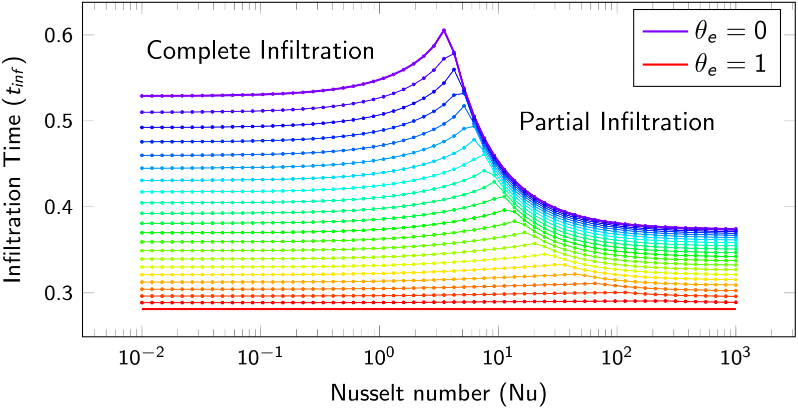
Stable propagation agrees with small-time solution, then clogs due to the boundary.

Numerical results



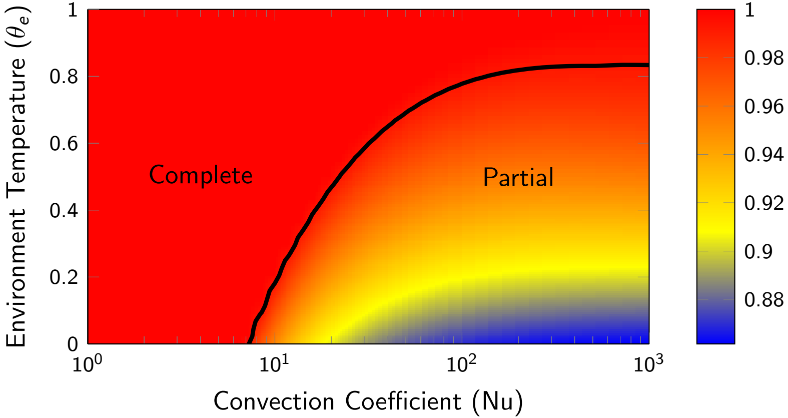
Unstable solution switches to the stable one on a short timescale, then clogs due to the boundary.

Time vs Nusselt number



The front stops later than in the isothermal infiltration case (🟥), the behaviour is non-monotonous (balance between clogging — infiltrating).

Complete vs Partial infiltration



Measuring the infiltrated height (colour) can be used to identify the convection coefficient, furthermore the infiltrated height is also correlated with the apparent density of the raft.

Conclusions and Future work

Conclusions:

- Developed a multiphase model for the infiltration of molten cryolite into a cold porous alumina.
- Investigated the relevant small overheat $1 - \theta \ll 1$ limit having an interesting type of Stefan condition coupling Darcy flow to heat equation.
- Similarity solution at small times yields nonuniqueness with one stable solution and nonexistence in certain regions of parameters.
- Late time simulations show either clogging or complete infiltration depending on the top boundary condition.

Future work:

- Modelling the disintegration of the raft (next stage of the industrial problem)
- Refine physics (dropping LTE, composition effects)

Thank you!

This work is supported by the EPSRC Centre For Doctoral Training in Industrially Focused Mathematical Modelling (EP/L015803/1) in collaboration with NORCE and Hydro Aluminium. Furthermore, this work is partly funded by SFI Metal Production, Centre for Research-based Innovation, 237738. Financial support from the Research Council of Norway and the partners of SFI Metal Production is gratefully acknowledged.

Csilla Kaszás, László I Kiss, Sándor Poncsák, and Jean-François Bilodeau.
Flotation and Infiltration of Artificial Alumina Rafts on the Surface of Molten Cryolite.
In *ICSOBA*, Quebec, 2016.

A. Mortensen, L. J. Masur, J. A. Cornie, and M. C. Flemings.
Infiltration of fibrous preforms by a pure metal: Part I. Theory.
Metallurgical Transactions A, 20(11):2535–2547, 1989.

G. G. Tsypkin.
Two-Valued Solutions in the Problem of Salt Precipitation during Groundwater Evaporation.
Fluid Dynamics, 40(4):593–599, 2005.